ELSEVIER
3 January 2002
PHYSICS LETTERS B
Physics Letters B 524 (2002) 185–191
www.elsevier.com/locate/npe

On the M5 and the AdS_7/CFT_6 correspondence

A.J. Nurmagambetov [1], I.Y. Park

Center for Theoretical Physics, Texas A&M University, College Station, TX 77843, USA

Received 1 November 2001; received in revised form 19 November 2001; accepted 19 November 2001
Editor: M. Cvetič

Abstract

The chiral primary operators of the $D = 6$ superconformal $(2,0)$ theory corresponding to 14 scalars of $N = 4$ $D = 7$ supergravity are obtained by expanding the world volume action for the M5-brane around an $AdS_7 \times S^4$ background. In the leading order, the operators take their values in the symmetric traceless representation of the $SO(5)$ R-symmetry group in consistency with the early conjecture on their structure based on the superconformal symmetry and Matrix-like model arguments.

PACS: 11.15.-q; 11.17.+y
Keywords: AdS/CFT; M5-brane; Supergravity

1. Introduction

The AdS/CFT correspondence [1–3] has revived the interest in superconformal theories and $AdS_{D-p} \times S^p$ configurations in supergravity (SUGRA) theories. During the past several years the correspondence between supergravity modes and super-Yang–Mills (SYM) operators was verified through different methods. (See Ref. [4] for an extensive list of references.) Many results have been obtained for AdS_5/CFT_4 correspondence. In the cases of AdS_4/CFT_3 and AdS_7/CFT_6 a relatively smaller number of papers were written.

The AdS_4 and AdS_7 geometry arise [5] in the large-N limit of N-coincident M2-branes [6,7] and M5-branes [8–10], respectively.[2] In this Letter we focus on the case of the AdS_7/CFT_6 correspondence. The structure of the CFT operators was obtained by analyzing the representations of superalgebra $Osp(8^*|4)$ [15–17]. Then the correspondence between CFT operators and supergravity modes can be established by comparing the various quantum numbers of their representations of $Osp(8^*|4)$. Based on such considerations it was conjectured in [18] that the chiral primaries of the $(2,0)$ CFT are scalar operators in the symmetric traceless reps. of the R-symmetry group $SO(5)$. This conjecture is in accordance with results [19,20] obtained from the Matrix-like DLCQ description of six-dimensional $(2,0)$ superconformal field theory as a quantum mechanics on the moduli space of instantons.

E-mail addresses: ajn@rainbow.physics.tamu.edu (A.J. Nurmagambetov), ipark@rainbow.physics.tamu.edu (I.Y. Park).

[1] Also at Institute for Theoretical Physics NSC KIPT, Kharkov, Ukraine.

[2] Covariant equations of motion for the M5-brane were obtained in [11] from the superembedding approach (see, e.g., [12], and [13] for recent reviews). Relations between different formulations were established in [14].

0370-2693/02
PII: S0370-2693(01)01375-2

Another way of matching various boundary CFT operators with the supergravity modes was proposed in [21]. The authors derived the SYM operators that are dual to the longitudinally polarized NS–NS two-form gauge field by expanding D3-brane action around an $AdS_5 \times S^5$ background.[3] In [22] this approach was applied to obtain the CFT operators that correspond to 20 scalar modes of the five-dimensional gauged supergravity. For the computation the Kaluza–Klein reduction [26,27] ansatz obtained in [28] was used.

Here following [21] and [22], we will consider the expansion of the Abelian M5-brane action around $AdS_7 \times S^4$ background. Although this geometry naturally arises in the large-N limit of N-coincident M5-branes, a non-Abelian action for such a system is still unknown. Therefore, in this Letter we consider the Abelian M5-brane probe propagating in the background of $(N-1)$ coincident M5-branes. In Section 2 we briefly review the structure of the non-linear ansatz for reduction of eleven-dimensional supergravity on S^4 and the equations of motions of seven-dimensional supergravity [29,30]. For our aim, consideration of the bosonic subsectors is sufficient. To simplify the calculations, in Section 3 we set seven-dimensional gauge fields to zero, which imposes additional constraints on the scalar sector of the $D=7$ supergravity. This, in turn, allows us to choose a diagonal parameterization for the scalar matrix. After substituting the ansätze for metric and target space gauge fields into the M5 action, we work out the CFT operators expanding the M5 action to linear order in the diagonal modes. Finally, we relax the zero setting of the gauge fields and obtain the CFT operators corresponding to the full set of the scalar fields. The last section contains our conclusions.

2. $D=11$ and $D=7$ SUGRA analysis

The starting point is the action of $D=11$ SUGRA [31]

$$S_{CJS} = \int d^{11}x \sqrt{-\hat{g}}\left[\widehat{R}(\hat{\omega}) + \cdots\right] - \int d^{11}x \sqrt{-\hat{g}}\, \frac{1}{2!4!} \widehat{F}^{(4)}_{\underline{\hat{m}}_1 \cdots \underline{\hat{m}}_4} \widehat{F}^{(4)\underline{\hat{m}}_1 \cdots \underline{\hat{m}}_4} - \int_{\mathcal{M}^{11}} \frac{1}{6} \hat{A}^{(3)} \wedge \widehat{F}^{(4)} \wedge \widehat{F}^{(4)}, \tag{1}$$

where the ellipses denotes the terms involving the Rarita–Schwinger field. The equation of motion for $A^{(3)}$ is

$$d\left(\hat{*}\widehat{F}^{(4)} - \frac{1}{2}\hat{A}^{(3)} \wedge \widehat{F}^{(4)}\right) = 0, \tag{2}$$

which can be viewed as the first order Bianchi identity for the dual field strength [32,33]

$$d\widehat{F}^{(7)} = 0, \qquad \widehat{F}^{(7)} = d\hat{A}^{(6)} + \frac{1}{2}\hat{A}^{(3)} \wedge \widehat{F}^{(4)}. \tag{3}$$

The equations of motion for $N=4$ $D=7$ $SO(5)$ gauged SUGRA [34] can be obtained from the $D=11$ SUGRA by the use of non-linear Kaluza–Klein S^4 reduction ansatz presented in [29,30]. In the notation of [35], it is given by

$$d\hat{s}_{11}^2 = \tilde{\Delta}^{1/3} ds_7^2 + g^{-2}\tilde{\Delta}^{-2/3} T^{-1}_{ab} D\mu^a D\mu^b, \tag{4}$$

$$\begin{aligned}\widehat{F}^{(4)} = {} & \frac{1}{4!}\epsilon_{a_1 \cdots a_5}\Big[-\frac{1}{g^3} U \tilde{\Delta}^{-2} \mu^{a_1} D\mu^{a_2} \wedge \cdots \wedge D\mu^{a_5} \\ & + \frac{4}{g^3}\tilde{\Delta}^{-2} T^{a_1}{}_b DT^{a_2}{}_c \mu^b \mu^c D\mu^{a_3} \wedge \cdots \wedge D\mu^{a_5} \\ & + \frac{6}{g^2}\tilde{\Delta}^{-1} F^{(2)a_1 a_2} \wedge D\mu^{a_3} \wedge D\mu^{a_4} T^{a_5}{}_b \mu^b\Big] \\ & - T_{ab} * C^{(3)a} \mu^b + \frac{1}{g} C^{(3)}_a D\mu^a,\end{aligned} \tag{5}$$

$$\begin{aligned}\widehat{F}^{(7)} = {} & -gU\epsilon_{(7)} - g^{-1}\left(T^{-1}_{ab} * DT_{bc}\right) \wedge \left(\mu^c D\mu^a\right) \\ & + \frac{1}{2} g^{-2} T^{-1}_{ac} T^{-1}_{bd} * F^{(2)ab} \wedge D\mu^c \wedge D\mu^d \\ & + g^{-4}\tilde{\Delta}^{-1} T_{ab} C^{(3)a} \mu^b \wedge W \\ & - \frac{1}{6} g^{-3}\tilde{\Delta}^{-1} \epsilon_{abcde} * C^{(3)f} T^a{}_f T^b{}_g \mu^g \\ & \wedge D\mu^c \wedge D\mu^d \wedge D\mu^e.\end{aligned} \tag{6}$$

Here

$$\begin{aligned} & U \equiv 2T_{ab}T_{bc}\mu^a\mu^c - \tilde{\Delta} T_{aa}, \quad \tilde{\Delta} \equiv T_{ab}\mu^a\mu^b, \\ & W = \frac{1}{4!}\epsilon_{a_1 \cdots a_5}\mu^{a_1} D\mu^{a_2} \wedge \cdots \wedge D\mu^{a_5}, \\ & F^{(2)}_{ab} = dA^{(1)}_{ab} + gA^{(1)}_{ac} \wedge A^{(1)c}{}_b, \\ & DT_{ab} = dT_{ab} + gA^{(1)}{}_a{}^c T_{cb} + gA^{(1)}{}_b{}^c T_{ac}, \\ & \mu^a\mu^a = 1, \qquad D\mu^a = d\mu^a + gA^{(1)}_{ab}\mu^b, \end{aligned} \tag{7}$$

[3] Further related discussions can be found in [23–25].

where $A^{(1)}_{ab}$ are the 10 gauge fields of $N=4$ $D=7$ gauged supergravity. In (4)–(7) $\epsilon_{(7)}$ is the volume form on the seven-dimensional space–time and T_{ab} is a symmetric unimodular matrix of scalars in the 14′ representation of $SO(5)$ which admits the following representation

$$T_{ab} = \left(e^S\right)_{ab}, \qquad \mathrm{Tr}\, S_{ab} = 0. \tag{8}$$

Substitution of the ansatz for $\widehat{F}^{(4)}$ and $\widehat{F}^{(7)} = \hat{*}\widehat{F}^{(4)}$ into the Bianchi identity for $\widehat{F}^{(4)}$ and $D=11$ equation of motion (2) leads to the following $D=7$ equations of motion

$$D\left(T_{ab} * C^{(3)b}\right) = F^{(2)}_{ab} \wedge C^{(3)b}, \tag{9}$$

$$H^{(4)}_a = gT_{ab} * C^{(3)b} + \frac{1}{8}\epsilon_{ab_1\cdots b_4} F^{(2)b_1b_2} \wedge F^{(2)b_3b_4}, \tag{10}$$

with $H^{(4)a} \equiv DC^{(3)a} = dC^{(3)a} + gA^{(1)}{}_b{}^a \wedge C^{(3)b}$,

$$\begin{aligned} &D\left(T^{-1}_{ab}T^{-1}_{cd} * F^{(2)ac}\right) \\ &\quad = -2gT^{-1}_{a[b} * DT_{d]a} - \frac{1}{2g}\epsilon_{a_1\cdots a_3bd} F^{(2)a_1a_2} \wedge H^{(4)a_3} \\ &\qquad + \frac{3}{2g}\delta^{b_1\cdots b_4}_{a_1a_2bd} F^{(2)a_1a_2} \wedge F^{(2)}_{b_1b_2} \wedge F^{(2)}_{b_3b_4} \\ &\qquad - C^{(3)}_b \wedge C^{(3)}_d, \end{aligned} \tag{11}$$

$$\begin{aligned} &D\left(T^{-1}_{ab} * DT_{bc}\right) \\ &\quad = 2g^2(2T_{ab}T_{bc} - T_{bb}T_{ac})\epsilon_{(7)} \\ &\qquad + T^{-1}_{ad}T^{-1}_{be} * F^{(2)de} \wedge F^{(2)b}{}_c + T_{cb} * C^{(3)b} \wedge C^{(3)}_a \\ &\qquad - \frac{1}{5}\delta_{ac}\left[2g^2\left(2T_{bd}T_{bd} - 2(T_{bb})^2\right)\epsilon_{(7)} \right. \\ &\qquad\qquad + T^{-1}_{bd}T^{-1}_{ef} * F^{(2)df} \wedge F^{(2)eb} \\ &\qquad\qquad \left. + T_{bd} * C^{(3)b} \wedge C^{(3)d}\right]. \end{aligned} \tag{12}$$

These equations, which are the bosonic part of the field equations of the seven-dimensional supergravity, will be relevant for our discussions below.

3. CFT operators from the M5-brane world volume action

Now let us calculate the CFT operators by expanding the M5-brane action in the $AdS_7 \times S^4$ background. To be concrete, we restrict our attention to the CFT operators that correspond to the SUGRA scalar only. The conformal dimension of these fields is equal to $\triangle = 2$ (see, e.g., [15–19]). Below we will, following [22], restrict to the subsectors of the scalar matrix T_{ab}. The full case is discussed at the end of this section.

The subsectors we consider are obtained by setting the gauge fields $A^{(1)}_{ab}$ and $C^{(3)}_a$ to zero. Then, Eqs. (9)–(12) reduces to

$$T^{-1}_{a[b} * dT_{d]a} = 0, \tag{13}$$

$$\begin{aligned} &d\left(T^{-1}_{ab} * dT_{bc}\right) \\ &\quad = 2g^2\left[(2T_{ab}T_{bc} - T_{bb}T_{ac}) \right. \\ &\qquad\qquad \left. - \tfrac{1}{5}\delta_{ac}\left(2T_{bd}T_{bd} - 2(T_{bb})^2\right)\right]\epsilon_{(7)}. \end{aligned} \tag{14}$$

Therefore, this setting allows one to choose a diagonal parameterization [30] for the matrix T_{ab}:

$$T_{ab} = \mathrm{diag}(X_1, \ldots, X_5), \qquad \prod_{a=1}^{5} X_a = 1 \tag{15}$$

and

$$X_a = \exp\left(-\frac{1}{2}\vec{b}_a \cdot \vec{\phi}\right). \tag{16}$$

Here $\vec{\phi}$ is the vector defining four independent scalars appearing in the reduction from M^{11} to $AdS_7 \times S^4$ and $\vec{b}_a$ are the weight vectors of the fundamental reps. of $SL(5,R)$ which have the following properties,

$$\vec{b}_a \cdot \vec{b}_b = 8\delta_{ab} - \frac{8}{5}, \qquad \sum_a \vec{b}_a = 0, \\ \sum_a (\vec{u} \cdot \vec{b}_a) \cdot \vec{b}_a = 8\vec{u} \tag{17}$$

for an arbitrary vector $\vec{u}$. [4]

[4] The explicit representations for the $\vec{b}_a$'s are as follows:

$$\vec{b}_1 = \left(2, \frac{2}{\sqrt{3}}, \frac{2}{\sqrt{6}}, \frac{\sqrt{2}}{\sqrt{5}}\right), \qquad \vec{b}_2 = \left(-2, \frac{2}{\sqrt{3}}, \frac{2}{\sqrt{6}}, \frac{\sqrt{2}}{\sqrt{5}}\right), \\ \vec{b}_3 = \left(0, -\frac{4}{\sqrt{3}}, \frac{2}{\sqrt{6}}, \frac{\sqrt{2}}{\sqrt{5}}\right), \qquad \vec{b}_4 = \left(0, 0, -\sqrt{6}, \frac{\sqrt{2}}{\sqrt{5}}\right), \\ \vec{b}_5 = \left(0, 0, 0, -\frac{4\sqrt{2}}{\sqrt{5}}\right).$$

After reconstruction of the Lagrangian and the equations of motions for the scalar fields, the n-point functions of the CFT operators can be computed, as discussed in [22], by use of the formulae in [36,37].

Substituting the diagonal parameterization (15) into the metric ansatz (4) and expanding it in linear order of $\vec{\phi}$, we have

$$ds_{11}^2 \simeq \left(1 - \frac{1}{6}\sum_a (\mu^a)^2 \vec{b}_a \cdot \vec{\phi}\right) ds_7^2 + g^{-2}\left(1 + \frac{1}{3}\sum_c (\mu^c)^2 \vec{b}_c \cdot \vec{\phi}\right) \times \sum_a \left(1 + \frac{1}{2}\vec{b}_a \cdot \vec{\phi}\right)(d\mu^a)^2. \tag{18}$$

To make the $SO(5)$ covariance manifest one can rewrite (18) in a coordinate system of Cartesian type, (x^i, x^a),

$$\mu^a = \frac{x^a}{r}, \quad r^2 = (x^a)^2. \tag{19}$$

Note that g is the inverse radius of the S^4, i.e., $g^{-1} = R$.

The space–time metric of BPS p-brane configurations has the form of (see, e.g., [5,38])

$$ds^2_{p\text{-brane}} = H^{-\frac{2}{p+1}}(dx^i)^2 + H^{\frac{2}{D-p-3}}(dx^a)^2, \tag{20}$$

$$H = 1 + \left(\frac{R}{r}\right)^{D-p-3}, \tag{21}$$

where the coordinates, x^i, are the brane coordinates and the coordinates, x^a, are transverse to the brane with $r^2 \equiv (x^a)^2$. In the near horizon region $r \ll R$ this metric simplifies to the geometry of an $AdS_{p+2} \times S^{D-p-2}$

$$ds^2 = \left(\frac{r}{R}\right)^{\frac{2(D-p-3)}{p+1}}(dx^i)^2 + \left(\frac{R}{r}\right)^2 (dx^a)^2. \tag{22}$$

For the M5 case the near-horizon region is $AdS_7 \times S^4$, with the metric given by

$$ds^2 = \left(\frac{r}{R}\right)(dx^i)^2 + \left(\frac{R}{r}\right)^2 (dx^a)^2. \tag{23}$$

Using this background metric, (18) can be rewritten as

$$ds_{11}^2 = grf\sum_i (dx^i)^2 + \frac{1}{g^2 r^2}\sum_{a,b=1}^{5} g_{ab}\, dx^a\, dx^b, \tag{24}$$

with

$$f = 1 - \frac{1}{6r^2}\sum_a (x^a)^2 \vec{b}_a \cdot \vec{\phi}, \tag{25}$$

$$g_{ab} = \delta_{ab} + \frac{1}{2}\vec{b}_a \cdot \vec{\phi}\delta_{ab} - \frac{1}{2r^2}\vec{b}_a \cdot \vec{\phi} x_a x_b + \frac{1}{3r^4}\sum_c (x^c)^2 \vec{b}_c \cdot \vec{\phi}\delta_{ab} - \frac{1}{2r^4}\sum_c (x^c)^2 \vec{b}_c \cdot \vec{\phi} x_a x_b. \tag{26}$$

There are additional terms coming from (18) which are of second order in $\vec{\phi}$, therefore, neglected.

Finally, we expand the action for the M5 [8–10]

$$S = -\int d^6\xi \left[\sqrt{-\det(\hat{g}_{mn} + i\widehat{H}^*_{mn})} + \frac{\sqrt{-\hat{g}}}{4\sqrt{-\widehat{(\partial a)^2}}}\widehat{H}^{*mn}\widehat{H}_{mnr}\partial^r a\right] + \int_{\mathcal{M}^6} \hat{A}^{(6)} + \frac{1}{2}db^{(2)} \wedge \hat{A}^{(3)} \tag{27}$$

around the background defined by (24) in the small velocities approximation [38]. In order to do that we need to find the explicit forms of the $\hat{A}^{(6)}$ and $\hat{A}^{(3)}$ gauge fields from the expressions of their field strengths, (5) and (6). After some algebra one can derive the following equations,

$$\hat{A}^{(3)} = -\frac{1}{3!g^3}\epsilon_{\alpha_1\cdots\alpha_4}\frac{X_\beta \delta_{\beta\alpha_4}\mu^{\alpha_4}}{\mu^0\tilde{\Delta}}d\mu^{\alpha_1}\cdots d\mu^{\alpha_3} - \frac{1}{3!g^3}\epsilon_{\alpha_1\cdots\alpha_4}\frac{1}{\mu^0(1+\mu^0)^2}\mu^{\alpha_1}d\mu^{\alpha_2}\cdots d\mu^{\alpha_4}, \tag{28}$$

$$\hat{A}^{(6)} = -\frac{1}{2g}\left(X_a^{-1} * dX_a (\mu^a)^2\right), \tag{29}$$

where we have split the index $a = 0, 1, \ldots, 4$ into the set of $(0, \alpha)$. Note that as in [22] these are on-shell results because they hold only up to the equation of motion, (14). However, the on-shell results are sufficient for our purpose.

The small velocity expansion [5] leads to

$$S \approx -\int d^6\xi \; -\frac{g^3 r}{2}\sum_a (x^a)^2 \vec{b}_a \cdot \vec{\phi} + \frac{1}{2}\sum_{ab}\left(\frac{1}{2}\vec{b}_a \cdot \vec{\phi}\delta_{ab} - \frac{1}{2r^2}\vec{b}_a\vec{\phi}x_a x_b - \frac{1}{3r^2}\sum_c (x^c)^2 \vec{b}_c \cdot \vec{\phi}\delta_{ab} + \cdots\right) \times \partial_m x^a \partial^m x^b + \cdots, \tag{30}$$

where we have omitted the terms of higher order in $\vec{\phi}$ or derivatives (of $\vec{\phi}$ and x^a as well).

Several remarks are in order concerning how to obtain (30). The general form of the expansion is

$$S \approx \int d^6\xi \mathcal{L}^{(0,0)} + \mathcal{L}^{(0,1)} + \mathcal{L}^{(1,0)} + \mathcal{L}^{(1,1)} + \cdots. \tag{31}$$

The superscript index, (p,q), indicates the order of $\vec{\phi}$ and the number of derivatives acting on them and x^a, respectively. Now we will prove that there are no other terms of the type $(1,0)$ than those we have already given in (30). To this end, note that the induced metric on the M5 worldvolume, which corresponds to the ansatz (4), has the following form

$$\hat{g}_{mn} = \tilde{\Delta}^{1/3}\left(g_{mn} + \tilde{\Delta}^{-1}T^{-1}_{ab} D_m \mu^a D_n \mu^b\right). \tag{32}$$

The action for the M5 also involves the inverse worldvolume metric $\hat{g}^{mn}$, which can be shown to be

$$\hat{g}^{mn} = \tilde{\Delta}^{-1/3}\left(g^{mn} - \frac{\tilde{\Delta}^{-1}T^{-1}_{ab} D^m \mu^a D^n \mu^b}{1 + \tilde{\Delta}^{-1}T^{-1}_{ab} D_m \mu^a D^m \mu^b}\right). \tag{33}$$

Up to the terms of the order $(2,0)$ Eqs. (32) and (33) can be written as $\hat{g}_{mn} \approx \tilde{\Delta}^{1/3} g_{mn}$ and $\hat{g}^{mn} \approx \tilde{\Delta}^{-1/3} g^{mn}$, respectively. The leading terms in $H^{(3)}$, which come from the first line of Eq. (27), are given by [39]

$$S_H \approx \int d^6\xi \sqrt{-\hat{g}} \times \left[\frac{1}{4!}\widehat{H}_{mnp}\widehat{H}^{mnp} + \frac{1}{8\widehat{\partial a \partial a}}\partial_m a\left(\widehat{H}^{mnl} - \widehat{H}^{*mnl}\right) \times \left(\widehat{H}_{nlp} - \widehat{H}^{*}_{nlp}\right)\partial^p a + \cdots\right]. \tag{34}$$

They do not contribute to the $(1,0)$ part because (34) has the "weight" $\tilde{\Delta}^0$ that only contributes to the $(0,0)$, $(2,0)$ and higher order in $\vec{\phi}$ with or without derivatives. As for the WZ terms, it is clear, from (29), that there is no contribution to the $(1,0)$ type terms from the $\hat{A}^{(6)}$ part. A straightforward calculation also shows that the contributions of the second term in the WZ part of the action are solely to $(0,0)$, $(2,0)$ and higher order terms, which completes our proof.

For the subsectors given by (15) and (16) we have achieved the goal because the CFT operator has appeared as the coefficient of $\vec{\phi}$. The coordinates x^a are transverse to the M5 worldvolume and they are the ones that are identified with the scalars Φ^a of the on-shell $(2,0)$ (ultrashort) supermultiplet.

For the full sectors one should keep the fields, $A^{(1)}_{ab}$ and $C^{(3)}_a$. After finding the complete ansatz for $\hat{A}_3$ and $\hat{A}_6$ and substituting them into the M5-brane action, one again only keeps the terms linear in S_{ab}. [6] Finally one should set all the supergravity modes to zero after taking the derivative with respect to S_{ab}: it is not difficult to see that the terms that involve $A^{(1)}_{ab}$ and $C^{(3)}_a$ will not be relevant for the final result. Therefore, we deduce from (30) the relevant part of the action through the following chain of relations

$$\begin{aligned} S &\approx -\int d^6\xi \; -\frac{g^3 r}{2}\sum_a (x^a)^2 \vec{b}_a \cdot \vec{\phi} \\ &= -\int d^6\xi \, g^3 r \sum_a (\Phi^a)^2\left[1 - \frac{1}{2}\vec{b}_a \cdot \vec{\phi} - 1\right] \\ &\approx -\int d^6\xi \, g^3 r \sum_a \left[e^{-\frac{1}{2}\vec{b}_a\vec{\phi}}(\Phi^a)^2 - (\Phi^a)^2\right] \\ &= -\int d^6\xi \, g^3 r \sum_{ab}\left[\left(e^S\right)_{ab}\Phi^a\Phi^b - (\Phi^a)^2\right], \end{aligned}$$

[5] We have used $\det M = \exp(\mathrm{Tr}\ln M)$, which in turn implies

$$\det(1+M)^{1/2} = 1 + \tfrac{1}{2}\mathrm{Tr}\, M - \tfrac{1}{4}\left[\mathrm{Tr}\, M^2 - \tfrac{1}{2}(\mathrm{Tr}\, M)^2\right] + \cdots.$$

[6] S_{ab} appeared in (8).

which implies

$$S \approx -\int d^6\xi\, g^3 r \sum_{ab} (\Phi^a \Phi^b) S_{ab}. \qquad (35)$$

In the boundary region, $r \to \infty$, $S_{ab} \propto r^{-1}$ and, therefore, the boundary condition can be chosen as

$$S_{ab|_{b.c.}} = \frac{1}{r} S^0_{ab}. \qquad (36)$$

Taking the trace constraint on S_{ab} into account we obtain the CFT operator,

$$\mathcal{O}^{ab} = \left(\Phi^a \Phi^b - \tfrac{1}{5}\delta^{ab}\Phi^c \Phi_c\right) + \cdots. \qquad (37)$$

4. Conclusions

Substituting the non-linear ansatz for the eleven-dimensional metric and gauge fields into the Abelian M5-brane action and expanding it around an $AdS_7 \times S^4$ background we have obtained the CFT operators that correspond to 14 scalars of $N = 4$ $D = 7$ supergravity. The leading terms of the operators are in the symmetric traceless representation of the $SO(5)$ R-symmetry group. Therefore, our result is consistent, in the leading order, with the conjecture based on the superconformal symmetry and Matrix-like model arguments.

However, the CFT operators have subleading terms as well that include, e.g., the $(2,0)$ CFT scalar fields and their derivatives. Appearance of such terms has been discussed in [40] in the context of type IIB supergravity on $AdS_5 \times S^5$. As noted in [22] the subleading terms appearing in the CFT operator could be viewed as in accordance with claim of [40] that supergravity modes are dual to the "extended" chiral primary operators. Or/and there could be some field redefinitions on the CFT side such as the one discussed in [41]. The interesting problem, therefore, is to compute the n-point correlators for scalar supergravity modes propagating on AdS_7 by the use of non-linear reduction ansatz [7] and to check explicitly this observation.

Another problem one can consider is to extend the results obtained here to another class of CFT operators that correspond to other supergravity modes and to compare with the results of [44] based on the primary superfields considerations.

[7] Two- and three-point correlators of the $(2,0)$ CFT primaries have been computed in [42] in linear ansatz approximation. An advantage of using the non-linear ansatz was discussed in [43].

Acknowledgements

The authors are grateful to Ergin Sezgin and Chris Pope for interest in this work and encouragement. A.J.N. is thankful to Vladimir Akulov, Igor Bandos and Dmitri Sorokin for fruitful and illuminating discussions. The work of A.J.N. is supported in part by the NSF Grant PHY-0070964, by INTAS under a Call 2000 Project N254 and by the Ukrainian Ministry of Science and Education Grant N2.51.1/52-F5/1795-98. The work of I.Y.P. is supported by US Department of Energy under grant DE-FG03-95ER40917.

References

[1] J.M. Maldacena, Adv. Theor. Math. Phys. 2 (1998) 231;
J.M. Maldacena, Int. J. Theor. Phys. 38 (1999) 1113.
[2] S.S. Gubser, I.R. Klebanov, A.M. Polyakov, Phys. Lett. B 428 (1998) 105.
[3] E. Witten, Adv. Theor. Math. Phys. 2 (1998) 253.
[4] O. Aharony, S.S. Gubser, J. Maldacena, H. Ooguri, Y. Oz, Phys. Rep. 323 (2000) 183.
[5] G.W. Gibbons, P.K. Townsend, Phys. Rev. Lett. 71 (1993) 3754.
[6] E. Bergshoeff, E. Sezgin, P.K. Townsend, Phys. Lett. B 189 (1987) 75.
[7] E. Bergshoeff, E. Sezgin, P.K. Townsend, Ann. Phys. 185 (1988) 330.
[8] P. Pasti, D. Sorokin, M. Tonin, Phys. Lett. B 398 (1997) 41.
[9] I. Bandos, K. Lechner, A. Nurmagambetov, P. Pasti, D. Sorokin, M. Tonin, Phys. Rev. Lett. 78 (1997) 4332.
[10] M. Aganagic, J. Park, C. Popescu, J.H. Schwarz, Nucl. Phys. B 496 (1997) 191.
[11] P.S. Howe, E. Sezgin, Phys. Lett. B 394 (1997) 62;
P.S. Howe, E. Sezgin, P.C. West, Phys. Lett. B 399 (1997) 49.
[12] I. Bandos, P. Pasti, D. Sorokin, M. Tonin, D. Volkov, Nucl. Phys. B 446 (1995) 79.
[13] P.S. Howe, E. Sezgin, P.C. West, Aspects of Superembeddings, D.V. Volkov Memorial Volume, Lecture Notes in Physics, Vol. 509, Springer-Verlag, 1998, 64;
D. Sorokin, Phys. Rep. 329 (2000) 1;
D. Sorokin, hep-th/0105102.
[14] I. Bandos, K. Lechner, A. Nurmagambetov, P. Pasti, D. Sorokin, M. Tonin, Phys. Lett. B 408 (1997) 135.
[15] S. Minwalla, Adv. Theor. Math. Phys. 2 (1998) 781.
[16] O. Aharony, Y. Oz, Z. Yin, Phys. Lett. B 430 (1998) 87.
[17] R.G. Leigh, M. Rozali, Phys. Lett. B 431 (1998) 311.

[18] S. Minwalla, JHEP 9810 (1998) 002.
[19] N. Seiberg, Nucl. Phys. Proc. Suppl. 67 (1998) 158.
[20] O. Aharony, M. Berkooz, N. Seiberg, Adv. Theor. Math. Phys. 2 (1998) 119.
[21] S.R. Das, S.P. Trivedi, Phys. Lett. B 445 (1998) 142.
[22] I.Y. Park, A. Sadrzadeh, T.A. Tran, Phys. Lett. B 497 (2001) 303.
[23] I.Y. Park, Phys. Lett. B 468 (1999) 213.
[24] S.R. Das, S.P. Trivedi, JHEP 0102 (2001) 046.
[25] L. Rastelli, M. Van Raamsdonk, JHEP 0012 (2000) 005.
[26] P. Van Nieuwenhuizen, An introduction to simple supergravity and the Kaluza–Klein program, in: B.S. de Witt, R. Stora (Eds.), Relativity, Groups and Topology, Les Houches, Session XL, 1983, Elsevier, Amsterdam, 1984, p. 825.
[27] M.J. Duff, B.E.W. Nilsson, C.N. Pope, Phys. Rep. 130 (1986) 1;
D. Sorokin, V. Tkach, Sov. J. Part. Nucl. 18 (1987) 441;
C.N. Pope, Lectures on Kaluza–Klein Theory, http://faculty.physics.tamu.edu/pope/.
[28] M. Cvetič, H. Lu, C.N. Pope, A. Sadrzadeh, T.A. Tran, Nucl. Phys. B 586 (2000) 275.
[29] H. Nastase, D. Vaman, P. Van Nieuwenhuizen, Phys. Lett. B 469 (1999) 96;
H. Nastase, D. Vaman, P. Van Niewenhuizen, Nucl. Phys. B 581 (2000) 179;
H. Nastase, D. Vaman, Nucl. Phys. B 583 (2000) 211.
[30] M. Cvetič, S.S. Gubser, H. Lu, C.N. Pope, Phys. Rev. D 62 (2000) 086003;
M. Cvetič, H. Lu, C.N. Pope, Phys. Rev. D 62 (2000) 064028.
[31] E. Cremmer, B. Julia, J. Scherk, Phys. Lett. B 76 (1978) 409.
[32] I. Bandos, N. Berkovits, D. Sorokin, Nucl. Phys. B 522 (1998) 214;
D. Sorokin, hep-th/9806175.
[33] E. Cremmer, B. Julia, H. Lü, C.N. Pope, Nucl. Phys. B 535 (1998) 242.
[34] M. Pernici, K. Pilch, P. Van Nieuwenhuizen, Phys. Lett. B 143 (1984) 103.
[35] M. Cvetič, H. Lu, C.N. Pope, A. Sadrzadeh, T.A. Tran, Nucl. Phys. B 590 (2000) 233.
[36] D.Z. Freedman, S.D. Mathur, A. Matusis, L. Rastelli, Nucl. Phys. B 546 (1999) 96.
[37] W. Mueck, K.S. Viswanathan, Phys. Rev. D 58 (1998) 041901.
[38] P. Claus, R. Kallosh, J. Kumar, P.K. Townsend, A. Van Proeyen, JHEP 9806 (1998) 004.
[39] I. Bandos, A. Nurmagambetov, D. Sorokin, Nucl. Phys. B 586 (2000) 315;
P. Pasti, D. Sorokin, M. Tonin, Phys. Rev. D 55 (1997) 6292.
[40] G. Arutyunov, S. Frolov, Phys. Rev. D 62 (2000) 064016;
G. Arutyunov, S. Frolov, JHEP 0004 (2000) 017.
[41] F. Gonzalez-Rey, B. Kulik, I.Y. Park, M. Rocek, Nucl. Phys. B 544 (1999) 218.
[42] R. Corrado, B. Florea, R. McNees, Phys. Rev. D 60 (1999) 085011;
F. Bastianelli, R. Zucchini, Nucl. Phys. B 574 (2000) 107.
[43] H. Nastase, D. Vaman, hep-th/0004123.
[44] B. Eden, S. Ferrara, E. Sokatchev, hep-th/0107084;
S. Ferrara, E. Sokatchev, hep-th/0110174.